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Direct CP violation in semi-leptonic and leptonic decays

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We show that direct CP violation in semi-leptonic and leptonic decays can occur in multi-Higgs doublet extensions of the electroweak standard model with flavor changing neutral currents. For pion and lepton decays this CP violating effects cannot be constrained by experimental data since up to now the branching ratio of the decays π^- and μ^- have not been measured in laboratory.

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Recently it was pointed out by Kaplan [1,2] that the comparison between the polarizations of μ^+ from the decay of π^+ and of μ^- from the decay of π^- could be used in order to verify if CP is violated in the $\pi \to \mu \to e$ chain decay. Denoting by A_{π^+} and A_{π^-} the oscillation amplitudes for muons from π^+ and π^- respectively it was found from data [3] that [1]

$$-0.01 < A_{CP} \equiv \frac{A_{\pi^+} - A_{\pi^-}}{A_{\pi^+} + A_{\pi^-}} < 0.02. \tag{1}$$

If this asymmetry is confirmed in the future, *i.e.*, $A_{CP} \neq 0$, it means the existence of CP violation in pion and/or muon decays. Hence, we can ask ourselves what sort of models can produce them.

The goal of this work is to point out that multi-Higgs doublet extensions of the $SU(2)_L \otimes U(1)_Y$ model with flavor changing neutral currents (FCNC) in the Yukawa sector and CP violation through the flavor mixing matrix in the interactions with the vector bosons $W^\pm$ or, through the scalar sector, imply direct CP violation in semi-leptonic and leptonic decays. We also introduce a different way for counting the physical phases in the fermion mixing matrices. Although this way coincides with the usual one, it is more appropriate when there are flavor changing neutral currents in a given model. If there is CP violation but not FCNC the effects are proportional to the fermion mass and therefore negligible. This of course implies constraints coming from de neutral meson parameters, notwithstanding, since there are new mixing angles those constraints do not necessarily imply large mass for both neutral and charged scalars. For instance masses of the order of 150 GeV are still possible in models with similar effects to the present one [4].

In the context of the SM [5] the only source of CP violation is the phase in the mixing matrix $V_{\rm CKM}$ of the vector charged currents [6] or, if we enlarge the Higgs sector it is possible to implement spontaneous or explicit CP violation through the scalar exchange [7]. Here we will point out an effect which arises when a model has any kind of CP violation and also flavor changing neutral currents (FCNC).

In the electroweak standard model (ESM by short) based on the gauge symmetry $SU(2)_L \otimes U(1)_Y$ and with only one Higgs doublet, the Yukawa interactions in the quark sector are

$$-\mathcal{L}^q_Y = \bar{\psi}_L(\Gamma^d \Phi D'_R + \Gamma^u \tilde{\Phi} U'_R) + H.c., \tag{2}$$

with $\Phi = (\phi^+,\, \phi^0)^T$, $\tilde{\Phi} = i\tau^2\Phi^*$, and $\Gamma^{d,u}$ being arbitrary complex matrices (Yukawa couplings) in the flavor space, $\psi_L = (U',\ D')_L$ denotes the doublet of left-handed fields; D'_R and U'_R are gauge singlets; τ^2 is the Pauli matrix and primed fields denote symmetry eigenstates. After the spontaneous symmetry breaking the neutral component of the scalar doublet ϕ^0 is shifted: $\phi^0 = (v + H^0)/\sqrt{2}$; being v the vacuum expectation value (VEV) and H^0 a physical scalar field. Then, the Yukawa neutral interaction reads in the symmetry basis

$$-\mathcal{L}^q_Y = (\bar{U'}_L M^u U'_R + \bar{D'}_L M^d D'_R + H.c.)\left(1 + \frac{H^0}{v}\right), \tag{3}$$

with the quark mass matrices $M^q = v\,\Gamma^q/\sqrt{2}$. Next, we must diagonalize the quark mass matrices M^u, M^d by using biunitary transformations

$$V^{u\dagger}_L M^u V^u_R = \hat{M}^u,\quad V^{d\dagger}_L M^d V^d_R = \hat{M}^d, \tag{4}$$

with
$\hat{M}^u \equiv {\rm Diag}(m_u,\, m_c,\, m_t)$ and $\hat{M}^d \equiv {\rm Diag}(m_d\,, m_s\,, m_b)$. The physical (unprimed fields) states are related to the symmetry eigenstates as follows:

$$U'_L = V^u_L U_L,\ U'_R = V^u_R U_R,\ \ D'_L = V^d_L D_L,\ D'_R = V^d_R D_R. \tag{5}$$

With Eqs. (4) and (5) the Yukawa interactions in Eq. (3) become diagonal in the flavor space

$$-\mathcal{L}^q_Y = \left(\bar{U}_L \hat{M}^u U_R + \bar{D}_L \hat{M}^d D_R + H.c.\right)\left(1 + \frac{H^0}{v}\right). \tag{6}$$

It means that there are no flavor changing neutral currents since $\hat{M}^{u,d}$ are diagonal matrices. This also happens in the neutral currents coupled to the Z^0 gauge boson. In terms of the physical fields the lagrangian does not depend at all on the $V^{u,d}_R$ matrices (we will show here that this is not the case when we have FCNC) and the matrices V^u_L and V^d_L appear only as the combination $V_{\text{CKM}} = V^{u\dagger}_L V^d_L$ in the charged currents coupled to W^+: $\bar{U}_L\gamma^\mu V_{\text{CKM}} D_L$ with $U_L = (u,c,t)^T_L$ and $D_L = (d,s,b)^T_L$ being mass eigenstates and V_{CKM} being an arbitrary unitary matrix. Next, it is necessary to determine how many phases in V_{CKM} (for simplicity this matrix will be denoted hereafter simply by V) are measurable. In quantum mechanics only the relative phases are important. Therefore, we can redefine the phases of the physical fields [8],

$$\tilde{u}_{\alpha L} = e^{i\varphi(\alpha)} u_{\alpha L}, \quad \tilde{d}_{\beta L} = e^{i\varphi(\beta)} d_{\beta L}, \tag{7}$$

where $\varphi(q)$ are arbitrary real numbers. There are $2N$ of such quantities if there are N generations. Under the above transformations we have (after absorbing the phases we will forget the "tilde" in the fields)

$$\bar{U}_L\gamma^\mu V\, D_L \to \bar{U}_L\gamma^\mu V' D_L = \bar{U}_L\gamma^\mu F^u\, V\, F^{d\dagger} D_L, \tag{8}$$

where $F^u \equiv \text{Diag}(e^{i\varphi(u)}, e^{i\varphi(c)}, e^{i\varphi(t)}, \dots)$ and similarly for F^d. In general we can write $V'_{\alpha\beta} = e^{i[\varphi(\beta)-\varphi(\alpha)]}V_{\alpha\beta}$, where α and β denote an u-like and a d-like quark, respectively. A general $N\times N$ unitary matrix has N^2 parameters with $N(N-1)/2$ of them taken as Euler angles and the remaining ones being phases. We see that in the matrix V', $2N-1$ of these phases are not measurable. This comes out because we have $2N$ unmeasurable phases $\varphi(\beta)$ and $\varphi(\alpha)$ but in V' only the phase differences appear and there are $2N-1$ of such quantities (only a common phase transformation of all left-handed quarks leaves the elements of V invariant). Therefore, V has $N^2-(2N-1) = (N-1)^2$ parameters where $N(N-1)/2$ are rotation angles. So, the number of phases in V' is $(N-1)(N-2)/2$. Although this argument is correct we will consider a little modified one which seems to be more appropriate when the right-handed mixing matrices $V^{u,d}_R$ survive in the lagrangian density, like the case in which there are flavor changing neutral currents in the theory. However, it is still necessary to examine how this rephasing affects the remaining terms in the lagrangian.

In the ESM the fermion-neutral gauge boson interactions are flavor as well as helicity conserving. Thus, there is no effect of the rephasing of the left- handed fields. The Yukawa interactions, although they are flavor conserving, are not helicity conserving. However, it is possible to redefine the right-handed quarks exactly with the same phase as the corresponding left-handed ones and the Yukawa term remains unchanged too. That is,

$$\tilde{u}_{\alpha R} = e^{i\varphi(\alpha)} u_{\alpha R}, \quad \tilde{d}_{\beta R} = e^{i\varphi(\beta)} d_{\beta R}. \tag{9}$$

In terms of the tilded fields, the lagrangian in Eq. (6) is still diagonal, no trace of the phases introduced in Eqs. (7) and (9) survives.

As we said before, the Yukawa couplings $\Gamma^{u,d}$, or the mass matrices $M^{u,d}$, are arbitrary complex matrices. It means that they have $2N^2$ real parameters, or N^2 angles and N^2 phases. On the other hand, the matrices $V^{u,d}_{L,R}$ are unitary matrices that is, each one of them can have up to $N(N+1)/2$ phases. The matrices $\hat{M}^{u,d}$ are real and diagonal (with positive eigenvalues). It means that the N^2 phases of Γ^u (or Γ^d) must be absorbed in the $N(N+1) > N^2$ phases of V^u_L *plus* the phases of V^u_R. We see that V^u_L and V^u_R do not need to be each one of them general unitary matrix, since in this case they have together more phases than the number needed to diagonalize Γ^u. For instance, if we choose V^u_L to be a general unitary matrix, *i.e.*, with $N(N+1)/2$ phases, it is sufficient for V^u_R to have only $N(N-1)/2$ phases; or vice versa, if V^u_R is the general unitary matrix with $N(N+1)/2)$ phases, V^u_L has only $N(N-1)/2$ of them (similarly with the d-like sector).

In the context of the ESM or its extensions without FCNC both selections are indistinguishable. This can easily be seen as follows. In the mixing matrix of the charged currents coupled to the vector bosons $W^\pm$ only the product $V \equiv V^{u\dagger}_L V^d_L$ appears in the lagrangian. The matrices $V^{u,d}_R$ do not appear at all in the lagrangian. Thus, if we had chosen V^d_L (V^u_L) as the general unitary matrix, independently of the choice of V^u_L (V^d_L), the matrix V is itself a general unitary matrix with $N(N+1)/2$ phases. On the other hand, if we had chosen both V^u_L and V^d_L as being unitary matrices both with only $N(N-1)/2$ phases, the rest of the phases needed to get real and positive mass eigenvalues must be in the matrices $V^{u,d}_R$ and V has only $N(N-1)$ phases. The last number has to be equal or less than $N(N+1)/2$ which is the maximum number of phases allowed for an unitary matrix. Hence, $N(N-1) < N(N+1)/2$ for $N=2$; but the number of phases in V is again $N(N+1)/2$ for $N \geq 3$. If we use now the phase redefinition of the physical fields in Eqs. (7) and (9) the observable phases are as usual for $N \geq 3$ but for the case of $N=2$ we can have only one phase. It means that we can redefine not $2N-1=3$ phase fields but only $2N-2=2$. The matrix V^u_R has $N(N+1)/2$ or $N(N-1)/2$ phases, if the phases of V^u_L are $N(N-1)/2$ or

$N(N+1)/2$, respectively, (the same for V_R^d). The phases will be observable if the matrices $V_R^{u,d}$ do not disappear from the lagrangian as it is the case when the model has FCNC. Summarizing, for $N \geq 3$ we can always choose the number of phases equal to $(N-1)(N-2)/2$ in the interaction with the $W^\pm$ gauge boson. Anyway, there will be more phases in the $V_R^{u,d}$ mixing matrices that will be observable if these matrices survive in the lagrangian density.

In fact, this way of counting phases in the mixing matrices is important in models with additional interactions which are diagonal in the symmetry basis and have FCNC. For instance if gauge singlets like $\bar{\psi}_L\psi_R$ (or vectors like $\bar{\psi}_R\gamma^\mu\psi_R$) are allowed. This is the case in the context of the $SU(2)_L\otimes U(1)_Y$ model if new generations transform like a vector under the gauge symmetry.

With only one Higgs doublet there are no physical charged fields. However, in extensions of the ESM model with several Higgs doublets there are physical charged fields. If the model has no FCNC the interactions of these fields with the quarks have the form

$$\sum_i \left(\bar{U}_L V \hat{M}^d D_R \phi_i^+ - \bar{D}_L V^\dagger \hat{M}^d U_R \phi_i^-\right) + H.c., \quad (10)$$

and we see that the same mixing matrix V of the charged currents coupled to the vector boson $W^\pm$ appears also in these charged scalar-quark interactions. The same CP violating phases appear in both, the Yukawa interactions and in the charged currents coupled to the vector bosons. For two or more doublets the fields $\phi_i^\pm$ are still symmetry eigenstates, thus it will be possible to have CP violation if the mixing matrix in the scalar sector has nontrivial phases, but this is not relevant for the case considered here.

In a n-Higgs-doublet model with FCNC, the Yukawa term of the lagrangian in the quark sector is

$$-\mathcal{L}_Y = \sum_i \bar{\psi}_L \left(\Gamma_i^d \Phi_i\right) D_R' + H.c., \quad (11)$$

where $i = 1, \cdots n$; plus a similar term in U_R'. Here $\Gamma_i^{u,d}$ are again arbitrary complex matrices in the flavor space. After the spontaneous symmetry breaking we have $\phi_i^0 = (v_i + h_i^0)/\sqrt{2}$ and the h_i^0 fields being linear combinations of the physical neutral scalars ($h_i^0 = \sum_j O_{ij}H_j^0$); the mass matrices are diagonalized as follows

$$V_L^{u\dagger} \sum_i \frac{v_i}{\sqrt{2}}\Gamma_i^u V_R^u = \hat{M}^u,\; V_L^{d\dagger} \sum_i \frac{v_i}{\sqrt{2}}\Gamma_i^d V_R^d = \hat{M}^d. \quad (12)$$

The interaction terms with the neutral scalars are of the form

$$\sum_i \left(\bar{D}_L V_L^{d\dagger}\Gamma_i^d V_R^d D_R\right)\frac{h_i^0}{\sqrt{2}} + H.c.. \quad (13)$$

The matrices $V_{L,R}^d$ diagonalize $\sum_i v_i\Gamma_i^d$ but not $v_i\Gamma_i^d$ separately for each i; hence we have flavor changing neutral currents coupled to the neutral scalars. Notice that since Γ_i^d are arbitrary matrices $V_L^{d\dagger}\Gamma_i^d V_L^d$ have N^2 phases. We have no more freedom to redefine phases since we have already used it in absorbing the phases of the Cabibbo-Kobayashi-Maskawa matrix V, as discussed above. It means that even in the case of $N = 2$ generations we will have physical phases in the neutral currents via scalar exchange (more phases will appear if there are CP violating phases in the scalar propagators).

The charged Yukawa interactions are of the form

$$\sum_i \bar{U}_L V_L^{u\dagger}\Gamma_i^d V_R^d D_R \phi_i^- + H.c., \quad (14)$$

and the same number of phases of Eq. (13) survives here too. Since ϕ_i^+ are symmetry eigenstate fields we can rewrite Eq. (14) in terms of the mass eigenstates H_j^+ ($\phi_i^+ = \sum_j \mathcal{K}_{ij}H_j^+$)

$$\sum_j \bar{U}_L \mathcal{V}_j D_R H_j^- + H.c., \quad (15)$$

where we have defined

$$\mathcal{V}_{\alpha\beta;j} = \sum_i \left(V_L^{u\dagger}\Gamma_i^d V_R^d \mathcal{K}_{ij}\right)_{\alpha\beta} \quad (16)$$

with $\alpha = u, c, t$, $\beta = d, s, b$. Notice that the interactions in Eqs. (13) and (14) (or (15)) are not proportional to the quark masses; even if $\mathcal{K}_{ij}$ were complex matrices, there are N^2 phases in the matrix $\mathcal{V}$ in Eq.(16).

Concerning the charged leptons, they can be rotated like the $d-$like quarks in Eq. (5) but now with $V_{L,R}^l$ instead of $V_{L,R}^d$. In the lepton sector the Yukawa interactions are (with massless neutrinos)

$$-\mathcal{L}_Y^l = \sum_i \left(\bar{\nu}_L\, V_L^{l\dagger}\Gamma_i^l V_R^l\, l_R\phi_i^+ + \bar{l}_L\, V_L^{l\dagger}\Gamma_1^l V_R^l\, l_R\, \frac{h_i^0}{\sqrt{2}}\right) + H.c., \quad (17)$$

where we have redefined the neutrino fields so that there is no mixing in the current coupled to the vector bosons $W^\pm$. The mass matrix for the charged leptons $M^l = \sum_i (v_i/\sqrt{2})\Gamma_i^l$ is diagonalized as in the case of the quarks $V_L^{l\dagger}M^l V_R^l = \hat{M}^l$, with $\hat{M}^l = \mathrm{Diag}(m_e, m_\mu, m_\tau, \cdots)$. Hence, the unitary matrices $V_{L,R}^l$ diagonalize M^l but not $v_i\Gamma_i^l$ separately. Although we have redefine the neutrino fields in the charged currents coupled to the vector bosons $W^\pm$, the same is not possible in the interactions with $\phi_i^\pm$. Hence, even with massless neutrinos we cannot avoid, in general, to have FCNC mediated by scalars in the lepton sector as well. If we allow Γ_i^l to be general $N\times N$ complex matrices we have N^2 phases in the Yukawa interactions of the charged Higgs in the lepton sector.

The currents in Eq. (17) can be written in terms of the physical charged scalar:

$$\sum_j \left(\bar{\nu}_L \, \mathcal{V}^l_j \, l_R \, H^+_j + \bar{l}_L \, \mathcal{O}^l_j \, l_R \, H^0_j\right) + H.c., \tag{18a}$$

with

$$\mathcal{V}^l_{\alpha,\beta;j} = \sum_i \left(V^{l\dagger}_L \Gamma^l_i V^l_R \mathcal{K}_{ij}\right)_{\alpha\beta}, \tag{18b}$$

and

$$\mathcal{O}^l_{\alpha,\beta;j} = \sum_i \left(V^{l\dagger}_L \Gamma^l_i V^l_R O_{ij}\right)_{\alpha\beta}, \tag{18c}$$

where $\alpha, \beta = e, \mu, \tau$.

An important consequence of this kind of models is that they imply direct CP nonconserving processes. For instance, $\Delta S = 1$ processes like the $K^0_L \to 2\pi$ decay. In the ESM only penguin diagrams contribute to this sort of processes [9]. In the present context CP violation arises because of the interference of the amplitudes of the diagrams shown in Figs. 1 and 2. Similar effect exists in hyperon decays [10].

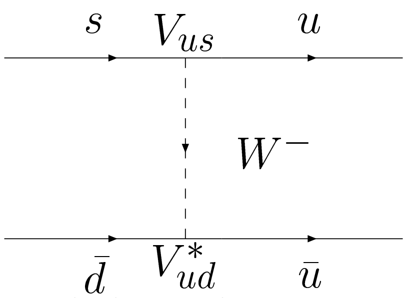

FIG. 1. Tree level vector boson W^- contribution to $K^0_L \to \pi^+\pi^-$.

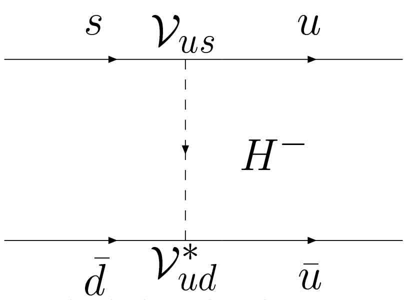

FIG. 2. Tree level charged scalar H^- contribution to $K^0_L \to \pi^+\pi^-$.

More interesting is the case of CP violation in semileptonic and leptonic decays. For instance, $\pi \to l\nu_l$ (particularly when $l = \mu$), $\tau \to \mu\nu\bar{\nu}$ and $\mu \to e\nu\bar{\nu}$ decays. Usually it is assumed that the π^+ decay conserves CP. For massless neutrinos the CP mirror image of the decay $\pi^+ \to \mu^+_{LH} + \nu_\mu$ is $\pi^- \to \mu^-_{RH} + \bar{\nu}_\mu$. In the first one the helicity of the muon is negative while in the second one it is positive. Positive pions come to rest then they decay as $\pi^+ \to \mu^+\nu_\mu$. Next, the muon after traveling some distance comes to rest and it decays as $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$. Events of the chain $\pi^- \to \mu^- \to e^-$ are not seen in this form since negative pions coming to rest in any material are attracted by a nucleus and captured at a rate too great for the decay be competitive. Hence, it follows that pions decaying in flight in vacuum are required for a CP test [11]. Similarly for the μ^- decay.

In models with multi Higgs doublets and FCNC the interference of the amplitudes in Figs. 3 and 4 implies CP violation in $\pi^\pm$ decays.

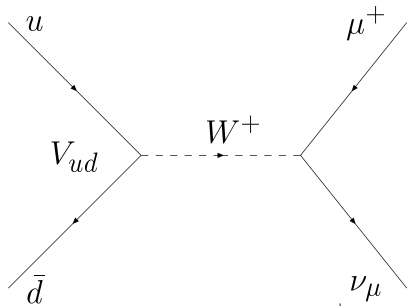

FIG. 3. Tree level charged scalar H^+ contribution to $\pi^+ \to \mu^+\nu_\mu$.

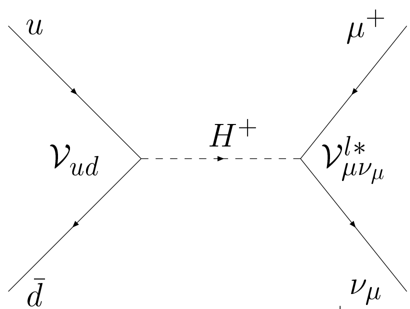

FIG. 4. Tree level charged scalar H^+ contribution to $\pi^+ \to \mu^+\nu_\mu$.

We can define the rate asymmetry

$$\Delta_\pi \equiv \frac{\Gamma_{\pi^+} - \Gamma_{\pi^-}}{\Gamma_{\pi^+} + \Gamma_{\pi^-}} = \frac{\kappa_\pi}{2\Gamma_{\pi^+} - \kappa_\pi}, \tag{19}$$

where

$$\begin{aligned} \kappa_\pi &\equiv \Gamma_{\pi^+} - \Gamma_{\pi^-} \approx \mathrm{Im}\left(V_{ud}\mathcal{V}_{ud}\mathcal{V}^{l*}_{\mu\nu_\mu}\right) \; \frac{G_F f^2_\pi m_\pi m^2_\mu}{8\pi m^2_H} \\ &\sim \mathrm{Im}\left(V_{ud}\mathcal{V}_{ud}\mathcal{V}^{l*}_{\mu\nu_\mu}\right) \times 10^8 \, \mathrm{s}^{-1}. \end{aligned} \tag{20}$$

We have assumed, since we are only interested here in an order of magnitude, that the decay constants for π^+ through axial-vector and pseudoscalar interactions are the same *i.e*, $f_\pi \approx 0.131$ GeV [12] and $m_H = 1000$ GeV in order to be sure that there are no large contributions to the $K - \bar{K}$ mass difference (although smaller masses are still allowed as it was mentioned above). Unfortunately we do not know what must be the value of Δ_π, since there is no a direct measure of the difference of the partial width of π^+ with respect to π^-. It is always measured Γ_{π^+} and it is assumed that the value for Γ_{π^-} is the same. However, if $\kappa \ll \Gamma_\pi$ we have

$$\Delta_\pi \approx \frac{\kappa_\pi}{\hbar}\frac{\tau_\pi}{2} \simeq 1.9 \times 10^{-7} \frac{\kappa_\pi}{\hbar}\,\mathrm{s}, \tag{21}$$

and we see that even if $\kappa_\pi/\hbar \approx 1\,\mathrm{s}^{-1}$ the CP asymmetry is of the order of 10^{-7} [13]. It means from Eq. (20) that

$$\mathrm{Im}(V_{ud}\mathcal{V}_{ud}\mathcal{V}^{l*}_{\mu\nu_\mu}) \approx 10^{-8}, \tag{22}$$

which is not an unreasonable value for the product of three matrix elements. However if $\kappa_\pi \sim O(\Gamma_\pi)$ we have $\Delta_\pi \approx 0.1$. The real value of Δ_π if different from zero may be in the middle of these values. We stress that Γ's matrices in Eqs. (18), in principle, are neither unitary nor hermitian, so the most general constraints come from perturbation theory: $|\Gamma|^2/4\pi < 1$. In fact we can saturate the value in Eq. (22) with $|V_{ud}\mathcal{V}^{l*}_{\mu\nu_\mu}|$ (or $|V_{ud}\mathcal{V}_{ud}|$) leaving $|\mathcal{V}_{ud}|$ (or $|\mathcal{V}^{l}_{\mu\nu_\mu}|$) arbitrary. From Figs. 1 and 2 we see that in the present model the contributions to $\epsilon^\prime_K$ at the tree level constrain $\mathcal{V}_{us}$; so, compatibility with data $Re\,(\epsilon^\prime_K/\epsilon_K) = (28 \pm 4.1) \times 10^{-4}$ from KTeV [14] and $(18.5 \pm 7.3) \times 10^{-4}$ from NA48 [15,16] can be obtained by choosing appropriately this matrix element.

Similar analysis can be done with the μ^+ and μ^- decays. In this case we can define in analogy with the Δ_π an asymmetry Δ_μ. However, it is not clear for us what is the relation between Δ_π and Δ_μ and the A_{CP} asymmetry in Eq. (1).

In the present model there are also contributions to $\epsilon^\prime_K$ coming from processes mediated by neutral Higgs bosons like the one shown in Fig. 5 and in the $K_L \to l\nu_l\pi$ decay because of the interference of $\bar{s} \to \bar{u}W^+ \to \bar{u}l\nu_l$ and $\bar{s} \to \bar{u}H^+ \to \bar{u}l\nu_l$.

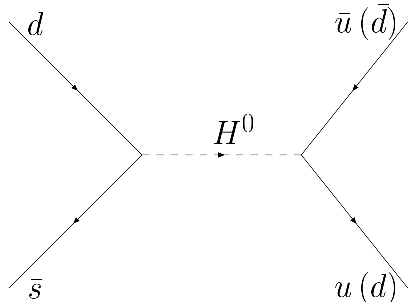

FIG. 5. Tree level contribution to $K^0 - \bar{K}^0$ mass difference and ϵ_K

In Ref. [13] the estimation of a possible CP violation effect in the $d\bar{u} \to W \to l^-\bar{\nu}$ vertex was obtained by considering new physics contributions to the $Wl\nu_l$ vertex. These contributions are proportional to q^2/Λ^2 with Λ a high energy scale, hence they are suppressed in pion decays. Multi-Higgs doublet extensions were not considered since it was supposed flavor conservation in the neutral-Higgs–fermion interactions and in this case the vertices are proportional to the fermion masses. In the present model this is not the case, we have flavor changing neutral currents in the scalar sector and it implies that the CP asymmetry can be, in principle, larger than 10^{-7}. Only experimental data can constrain this CP effect. The same happens with the μ^+ and μ^- modes. In fact, we stress that the only experiment which has data of $\pi^\pm$ and $\mu^\pm$ decays in flight is the SuperKamiokande. It means that although CP-violation cannot be by sure a dominant contribution to the atmospheric neutrino anomaly (for instnce, it cannot explain the azimuthal dependence) [17] an asymmetry in the $\pi\mu e$ decay chain of the order of 1-2% can be an important correction to the suppression of the neutrino fluxes since the neutrinos and antineutrinos would have different production. We would like to stress that the first available limit on CP violation in the $\pi \to \mu \to e$ decay chain is that of Ref. [1].

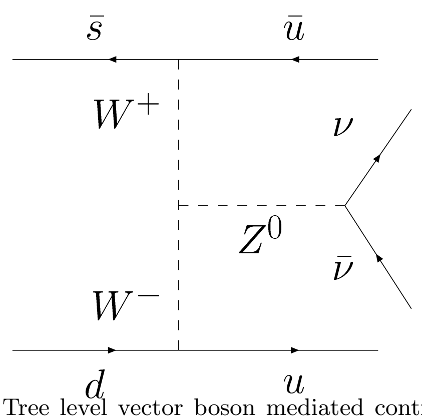

FIG. 6. Tree level vector boson mediated contribution to the $K_L \to \pi\nu\bar{\nu}$ decay.

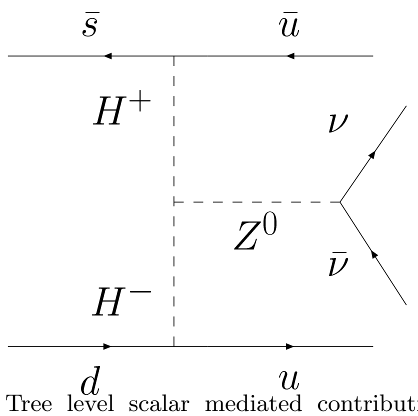

FIG. 7. Tree level scalar mediated contribution to the $K_L \to \pi\nu\bar{\nu}$ decay.

It is worth to make a remark with respect to the rare neutral kaon decays like $K_L \to \pi^0 e^+e^-$ [18] and $K_L \to \pi^0\nu\bar{\nu}$ [19]. Both decays in the standard model violate CP in leading order. In particular the decay $K_L \to \pi^0\nu\bar{\nu}$ is not only CP violating, but it is strongly dominated by direct CP violation without the potentially large 2γ mediated CP-conserving contributions which occur in the $K_L \to \pi^0 e^+e^-$ decay [20].

Denoting the CP-violating parameter $\bar{\eta}_{\pi\nu\bar{\nu}}$, it has been shown that $0.1 \lesssim \bar{\eta}_{\pi\nu\bar{\nu}} \lesssim 1$ [21], which is much larger than the corresponding $K \to \pi\pi$ parameters. Although in the standard model this decay has a branching ratio

$B(K_L \to \pi^0\nu\bar{\nu}) = 2.78 \times 10^{-11}$ [22] the experimental data give [12]

$$\frac{\Gamma(K_L \to \pi^0\nu\bar{\nu})}{\Gamma_{\text{total}}} < 4.3 \times 10^{-5}. \tag{23}$$

It means that this decay can be sensitive to new physics. In the standard model the main contributions to the decay $K_L \to \pi^0\nu\bar{\nu}$ come from penguin and box diagrams. On the other hand, in the present model this decay proceeds *via* diagrams like those in Figs. 6 and 7. The interference of both type of diagrams induces CP violating effects.

Independently of the CP issue, using the model independent ratio [23]

$$B(K_L \to \pi^0\nu\bar{\nu}) < 4.4 \times B(K^+ \to \pi^+\nu\bar{\nu}) \tag{24}$$

which is valid even if lepton flavor is not conserved and [12]

$$B(K^+ \to \pi^+\nu\bar{\nu}) = 4.2^{+9.7}_{-3.5} \times 10^{-10},$$

we obtain

$$B(K_L \to \pi^0\nu\bar{\nu}) < 2 \times 10^{-9}. \tag{25}$$

At first sight it appears that in the present model, unlike the $K_L \to \pi^0\nu\bar{\nu}$ decay which arises at the tree level, the decay $K^+ \to \pi^+\nu\bar{\nu}$ arises only at the 1-loop level. It means that the inequality in Eq. (24) which assumes only isospin relations can be evaded and $B(K_L \to \pi^0\nu\bar{\nu}) > B(K^+ \to \pi^+\nu\bar{\nu})$. Notice however that the effective interaction Lagrangian arisen from diagrams like those in Figs. 6 and 7 are not of the four-fermion form $(\bar{s}d)(\bar{\nu}\nu)$ but of the six-fermion form. For instance, the strength of the diagram in Fig. 7 is proportional to

$$\frac{(\mathcal{V}_j)^2_{ud} M_K}{M_H^4 m_Z^2}, \tag{26}$$

where M_H is a typical scalar mass and $M_K = 497.672$ MeV. The dimensionless ratio of the strength of the amplitude in Fig. 7 with respect to the four-fermion effective interaction Lagrangian in the standard model, denoted here by A_{SM}, is

$$R = \frac{M_K^6 (\mathcal{V}_j)^2_{ud}/M_H^4 M_Z^2}{A_{SM}} \tag{27}$$

where

$$A_{SM} = G_\mu \alpha(M_Z) 2\text{Im}C/\sqrt{2}\pi \sin^2\theta_W \approx 3.6 \times 10^{-11} \tag{28}$$

where we have used the values for the parameters in Eq. (28) given in Ref. [22]. Hence we have in Eq. (27)

$$R \approx 2 \times 10^5 \left(\frac{1\,\text{GeV}}{m_H}\right)^4 (\mathcal{V}_j)^2_{ud}. \tag{29}$$

We see that even a relatively light scalar $m > 80$ GeV gives a contribution which is 10^{-3} smaller than the standard model 1-loop contributions.

The decay $K_L \to \pi^0\nu\bar{\nu}$ was considered in two- and three-Higgs doublet models with and without FCNC in Ref. [24], in this sort of models even the penguin and box diagrams, the branching ratio for that decay is smaller than the standard model result and thus unmeasurable. However we stress that this is not necessarily the case for the pion decay. The asymmetry Δ_π defined in Eq. (19) can be measurable.

The decays $K_L \to \nu\bar{\nu}(\gamma)$ arise only if neutrino gets a mass [22], however the decay $K_L \to l^+l'^-$ proceeds at tree level through the $d\bar{s} \to H^0 \to l^+l'^-$ transitions. There will be also CP violation in another semileptonic decays as $B^0 \to X\nu_l l$; and also in $p\bar{p} \to l^\pm\nu X$ because of the interference of $p\bar{p} \to W^\pm X \to l^\pm\nu X$ with $p\bar{p} \to H^\pm X \to l^\pm\nu X$.

In the lepton sector the flavor violation effects via the neutral scalar exchange shown in Fig. 8 induce not only the usual muonium ($M \equiv \mu^+e^-$)–antimuonium ($\bar{M} \equiv \mu^-e^+$) transition [25] but also CP violation, this leaves this system closer to the neutral kaons [26]. Notice that there are scalar and pseudoscalar contributions to the $M \to \bar{M}$ transition [27]. If the $|\mathcal{V}^l_{\mu\nu_\mu}|$ matrix element is left arbitrary in the pion decay, the CP-violation neutral interactions given in Eqs. (18) can be large enough to be detected by comparing $M \to \bar{M}$ to $\bar{M} \to M$ conversions.

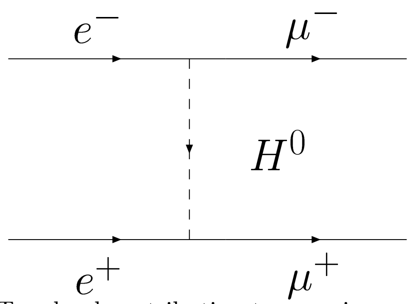

FIG. 8. Tree level contribution to muonium-antimuonium transition. There are scalar and pseudoscalar contributions.

Finally, we would like to stress that the features we have shown in this work can be implemented in other models with complicated Higgs sector and intermediate mass scales. An interesting possibility arises when, by imposing an appropriate discrete symmetry, the scalars coupled to the leptons are different from the scalars coupled to the quarks. In this case we have the so called "leptophilic" Higgs scalars since the VEV of the neutral scalars coupled to the leptons may not be necessarily of the same order of magnitude than the VEVs which give mass to the quarks and vector bosons [28].

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